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EXPERIMENTAL STUDY OF HEAT EXCHANGE IN A MODERATELY RAREFIED GAS

A. N. Kulev, A. M. Shestakov, S. F. Borisov, and Yu. G. Semenov

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An experimental study is performed of heat exchange in multiatomic gases at moderate Knudsen numbers in coaxial geometry. The data obtained are described well by the Liu–Lees theory of multiatomic gases.

The heat-exchange problem in rarefied gases has many practical applications, such as high-altitude aerodynamics, precision chemical technology, etc. On the other hand, comparison of experimental results on heat exchange obtained under conditions suitable for theoretical consideration of the problem provides valuable information on characteristics such as parameters of the intermolecular interaction potential and energy accommodation and tangential molecular momentum coefficients.

At present the character of heat exchange in the free molecular regime has been studied thoroughly [1, 2]. This is because it is possible to determine the energy accommodation coefficients characterizing energy exchange in the gas–solid system properly [3-5]. Heat exchange has been studied in greater detail in the continual limit [6, 7], since such studies are the major source of information on gas thermal conductivity coefficients. At the same time, experimental data for the region of transition between free molecular and continual regimes are few in number, especially for multiatomic gases.

There exist a large number of theoretical studies on heat exchange in planar geometry, based on solution of the Fourier equation using the temperature change condition on the boundary (see review [4]). However, such an approach does not permit description of the entire rarefaction range. This shortcoming can be eliminated only by solving Boltzmann's equation or a model thereof with substitution of proper boundary conditions.

Two studies are available [8, 9] in which heat exchange in monatomic gases was considered at arbitrary Knudsen number with the assumption of small temperature changes for the geometry met most often in practical applications – coaxial cylinders. Precise treatment of heat exchange in multiatomic gases is complicated by the necessity of introducing energy accommodation coefficients for the internal degrees of freedom. However, under conditions of complete molecular energy exchange with the surface such a description is possible by introducing into the solution some factor which considers the character of energy exchange in intermolecular interactions [10].

Below we will present results of experimental studies of thermal fluxes in certain mon- and multiatomic gases by the heated wire method at a temperature of about 300°K in the intermediate region (Kn = 0.01-1). Processing the data by the theory of [9], extended to multiatomic gases, allowed determination of the Eucken factor, which coincided with values avail-

A. M. Gor'kii Ural State University, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 4, pp. 579-585, October, 1985. Original article submitted September 4, 1984.

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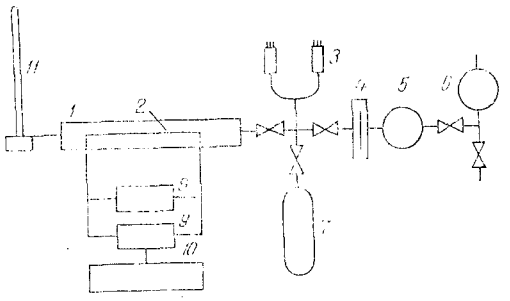

Fig. 1. Schematic diagram of experimental equipment.

able in the literature. This fact indicates the validity of the description of heat exchange in coaxial geometry proposed in [9].

Thermal fluxes were measured by the nonsteady state heated wire method using an RC-generator, described in detail in [11]. A schematic diagram of the experimental arrangement is shown in Fig. 1. The measurement cell was a stainless steel tube 1, on the axis of which a nickel wire 2 (200 ± 1) μm in diameter was held by ceramic inserts with centering orifices. The ratio of the diameter of the wire to the inner diameter of the outer cylinder was $(152 \pm 2)\cdot10^{-4}$. To decrease the influence of end effects, the measurement cell was about 1 m long.

The eccentricity of the wire location relative to the geometric axis of the external cylinder, caused by error in the external cylinder and centering washer construction, was estimated to be less than 600 μm. Such an eccentricity produces a thermal flux correction of about 0.05% [12].

The cell was first evacuated to a pressure of almost $1\cdot10^{-3}$ Pa by forevacuum pump 6 and diffusion pump 5 through nitrogen trap 4. The vacuum level was monitored by thermocouple and ionization pressure sensors 3. The gas to be studied was admitted into the cell from reservoir 7.

The nickel wire was heated by passage of an electrical current from power source 8 to a temperature 0.5-1°K above room temperature. At a certain point in time the heating was terminated and the wire was connected to RC-oscillator 9. The change in frequency of the RC-generator produced by cooling of the wire was recorded by frequency counter 10.

During the experiment the wire cooling rate was determined as a function of gas pressure. The pressure range used ensured achievement of the intermediate regime with respect to the wire diameter. The pressure was measured by mercury manometer 11 to an uncertainty not more than 1.5%. The uncertainty in cooling rate determination comprised about 1%. Experimental values of cooling rates as functions of pressure for various gases are shown in Fig. 2.

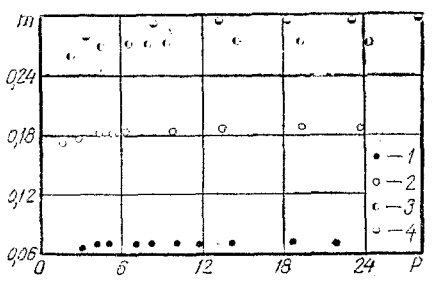

Fig. 2

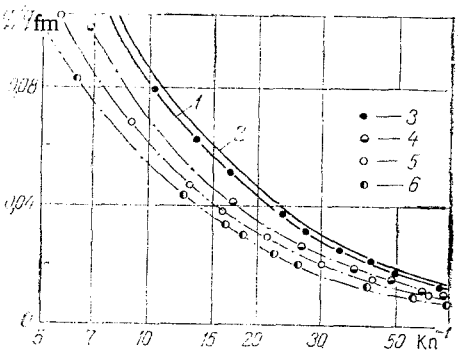

Fig. 3

Fig. 2. Cooling rate vs. pressure: 1) Xe; 2) CO_2; 3) NH_3; 4) N_2. m, 1/sec; P, mm Hg.

Fig. 3. Relative thermal flux vs. Knudsen number: 1) [9]; 2) [8]; 3) Xe; 4) N_2; 5) CO_2; 6) NH_3.

Agreement between theory and experiment was evaluated by comparing experimental and theoretical dependences of dimensionless thermal flux q/q_{fm} on Knudsen number Kn. Referring the thermal flux to the quantity q_{fm} is convenient and widely used in the literature, inasmuch as q_{fm} depends linearly on pressure. In this case, the ratio q/q_{fm} gives a clear picture of the features of transition into the intermediate regime.

The Knudsen number was calculated with the expression

$$\mathrm{Kn} = \frac{\eta}{r_1 \rho}\sqrt{\frac{\pi}{2RT_1}}. \tag{1}$$

The value of q/q_{fm} was determined from the experimental data following [3]:

$$\frac{q}{q_{fm}} = \frac{m - m_0}{m_{fm}}, \tag{2}$$

where m_{fm} is the cooling rate in the limit of free molecular heat exchange, calculated with the expression

$$m_{fm} = \alpha_1 P \frac{2(C_v + R/2)}{r_1 C_{Ni} \rho_{Ni}\sqrt{2\pi RT_1}}. \tag{3}$$

Here α_1 is the energy accommodation coefficient on the nickel wire surface. It was shown in [3] that for the gases of the present study on a nickel surface coated by films of unknown composition (which conditions were realized in the present experiments) within an accuracy of 1% the values of α_1 are equal to unity.

According to Eqs. (1)-(3), the uncertainties in determination of the quantities Kn and q/q_{fm} are 2 and 4%, respectively.

Relative thermal fluxes are shown as functions of the parameter Kn for the gases studied in Fig. 3. Also shown are theoretical functions obtained for a monatomic gas at $\alpha_1 = 1$. Curve 1 is a solution of the problem of heat exchange in a gas for coaxial geometry by the moment method [9]; curve 2 is an extrapolation of the solution of the BGK equation by the variation method [8] to the present experimental conditions.

As is evident from Fig. 3, there is a systematic divergence of the experimental results for the gases studied, apparently related to the participation in intermolecular exchange of internal degrees of molecular freedom, since energy exchange upon collision of molecules of these gases with the surface is total. On the other hand, it is evident from Fig. 3 that the data for Xe are described well by the theoretical dependence taken from [9] and obtained for the case of a monatomic gas for the condition of total energy exchange on the boundary. This fact permits us to conclude in favor of this theory and extend its results to heat exchange in multiatomic gases.

It can be proposed in analogy to [10] that the effect of the multiatomic nature of the molecules and their incomplete energy exchange upon collision with the surface may be described by introduction of some factor B before the second term in the denominator of the expression for specific thermal flux in Liu–Lees form [9]. In this case the dependence of thermal flux per unit area of a wire of radius r_1 upon Kn has the form:

$$q = \frac{\lambda(T_1 - T_2)}{r_1\left(\ln\frac{r_2}{r_1} + \frac{15}{4}B\,\mathrm{Kn}\right)}. \tag{4}$$

The expression for q_{fm} can be represented in the following form [2]:

$$q_{fm} = \frac{1}{2}C_V(1+\gamma)\rho\sqrt{\frac{RT_1}{2\pi}}(T_1 - T_2)\frac{F}{A}, \tag{5}$$

where A is a parameter characterizing molecular energy exchange on the surfaces of the wire and outer cylinder, defined by the expression

$$A = \frac{1}{\alpha_1} + \frac{r_1}{r_2}\left(\frac{1}{\alpha_2} - 1\right). \tag{6}$$

The quantity F is

$$F=\frac{\sqrt{1-\tau\frac{1-\alpha_1}{\alpha_1 A}}}{\sqrt{\frac{\alpha_1 A-\tau(1-\alpha_1)}{\alpha_1 A-\tau}}+\frac{1}{2}\left(1-\sqrt{\frac{\alpha_1 A-\tau(1-\alpha_1)}{\alpha_1 A-\tau}}\right)\times}\cdots$$

$$\cdots\rightarrow\frac{\sqrt{1-\tau\frac{1-\alpha_1}{\alpha_1 A}}}{\times\left[1-\frac{\frac{r_2}{r_1}\arccos\frac{r_1}{r_2}-\sqrt{1-(r_1/r_2)^2}}{\frac{\pi}{2}\left(\frac{r_2}{r_1}-\frac{r_1}{r_2}\right)}\right]}, \tag{7}$$

where τ is the dimensionless temperature difference, $\tau = (T_1 - T_2)/T_1$.

From Eqs. (4) and (5), considering the definition of the Knudsen number, Eq. (1), and the Eucken factor ($f = \lambda/C\gamma\eta$), we can obtain an expression in explicit form for the dimensionless thermal flux q/q_{fm} as a function of Kn:

$$\frac{q}{q_{fm}}=\frac{4f}{1+\gamma}\frac{A}{F}\frac{\mathrm{Kn}}{\ln\frac{r_2}{r_1}+B\frac{15}{4}\mathrm{Kn}}. \tag{8}$$

Since in the free molecular limit (Kn $\rightarrow \infty$) Eq. (8) must be equal to unity, for the quantity B we obtain the following expression:

$$B=\frac{16}{15}\frac{f}{1+\gamma}\frac{A}{F}. \tag{9}$$

In this expression the parameter A considers the effect of gas-surface interaction on heat exchange. The factor $\frac{16}{15}\frac{f}{1+\gamma}$ characterizes the effect of the multiatomic nature of the gas on heat transport in the gap. Using for f the expression proposed by Eucken [13], it is simple to show that for a monatomic gas B = 1, and for a biatomic gas B = 38/45. This result coincides with the conclusions of [10] and generalizes them to the case of an arbitrary multiatomic gas. It should be noted that for the case of low temperature differences ($\tau \rightarrow 0$), i.e., for a situation in which the Liu–Lees expression Eq. (4) is valid, the value F = 1. In particular, for the experimental data obtained F differs from unity by not more than 0.1%.

With consideration of Eq. (9), for the relative heat flux q/q_{fm} we finally obtain

$$\frac{q}{q_{fm}}=\frac{1}{1+\frac{4}{15}\frac{1}{B\,\mathrm{Kn}}\ln\frac{r_2}{r_1}}. \tag{10}$$

This expression was used to process the experimental data. Values of the quantity B for Xe, N_2, CO_2, and NH_3 and the random uncertainties in their determination are presented in Table 1. Also shown are values of f, calculated with Eq. (9), as well as values of the Eucken factor f_0, calculated from experimental data on thermal conductivity, viscosity, and specific heat [14].

In calculating f values for the gases studied with Eq. (9) the quantity A was taken equal to unity. It is evident that this will introduce an error into the calculation of f of no more than 1%, since for the concrete value of $r_1/r_2 = 0.0152$ the quantity A, according to

TABLE 1. Results of Processing Experimental Data

Gas	B	$\pm\Delta B$	f	$\pm\Delta f$	f_0	f_{rel}	$\pm\Delta f_{rel}$
Xe	0,992	±0,005	2,48	±0,02	2,60	—	—
N_2	0,824	±0,004	1,86	±0,01	1,95	1,95	±0,02
CO_2	0,728	±0,002	1,57	±0,01	1,68	1,65	±0,02
NH_3	0,629	±0,004	1,37	±0,01	1,46	1,44	±0,02

Eq. (6), is determined only by the value of α_1, which for the gases studies is within 1% of unity [3].

As is evident from Table 1, the values obtained for f are 4-5% less than the values of f_0. Such a divergence exceeds the random uncertainties in Δf determination presented in the table, and is due to the presence of systematic error in the quantity B, which according to Eq. (10) comprises 6-7%. To eliminate the systematic error, values of the Eucken factor for the multiatomic gases studied were calculated relative to xenon with the expression

$$f_{\text{rel}} = \frac{B}{B_{\text{Xe}}} f_{\text{Xe}} \frac{1+\gamma}{1+\gamma_{\text{Xe}}}. \tag{11}$$

It can easily be shown that the uncertainty in f_{rel} determination is defined by the uncertainty in the cooling rate and is at the level of 1-2%. Values of f_{rel} for multiatomic gases and their uncertainties are presented in Table 1.

From comparison of the f_0 and f_{rel} values presented in the table it is evident that deviations of f_{rel} from f_0 do not exceed 2% and lie within the limits of f_{rel} uncertainty.

Thus we may conclude that heat transport in multiatomic gases between coaxial cylinders in the intermediate regime is described well by the theoretical expressions obtained in [9], with correction for the multiatomic nature of the gases by Eq. (9).

It should also be noted that processing of the experimental data by the expressions of this theory permits (in the case of known Eucken factors) determination of the energy accommodation coefficients in the intermediate and continual regimes. Values of the accommodation coefficients calculated in this manner can differ significantly from values obtained in the free molecular regime, since change in gas pressure can lead to significant change in the quantity of molecules adsorbed on the surface and, consequently, to change in energy exchange between gas molecules and surface.

NOTATION

Kn, Knudsen number; q, specific thermal flux; q_{fm}, free molecular limit of specific thermal flux; r_1, r_2, wire and outer-cylinder radii; T_1, T_2, temperatures of wire and outer cylinder; R, universal gas constant; ρ, gas density; η, λ, coefficients of viscosity and thermal conductivity; m, cooling rate; m_0, cooling rate in vacuum; m_{fm}, cooling rate in free molecular heat exchange limit; α_1, α_2, energy accommodation coefficients on wire and outer cylinder surfaces; P, gas pressure; C_V, isochloric heat capacity of gas; C_{Ni}, ρ_{Ni}, specific heat and density of nickel; A, parameter characterizing molecular energy exchange with wire and outer-cylinder surfaces; B, factor considering effect of multiatomic nature of molecules on heat exchange; F, parameter characterizing equipment geometry; τ, dimensionless temperature change; f, Eucken factor; γ, adiabatic index.

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THERMAL DEFORMATION OF UNEVENLY HEATED JET HEAT EXCHANGERS

S. B. Koshelev, V. V. Kharitonov, and G. É. Ter-Avakimov

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Expressions are obtained for evaluating stresses and strains in an unevenly heated jet heat exchanger having the form of a metallic disk with a cellular structure.

Jet heat exchangers are widely used in different areas of technology [1-3]. Such heat exchangers may be significantly deformed at high thermal loads and temperature gradients, and their structural materials may experience intolerably high thermal stresses. The goal of the present study is to evaluate thermal stresses and strains in jet heat exchangers having the form of a metal disk of thickness H and diameter 2b with a cellular structure (Fig. 1) in the case of nonuniform axisymmetric heating. The cellular structure makes it possible to intensify heat transfer due to the finning effect and retains the bending stiffness of the exchanger.

Formulation of the Problem. The exact simultaneous solution of the differential equations of heat conduction (second order) and thermoelasticity (fourth order) is possible only in certain simple cases [4, 5]. Thus, various approximations of the theory of thermoelasticity are used in engineering calculations [4-7]. Below we represent the resulting strain $\omega(r)$ of the heated surface ($z = H/2$) approximately in the form of the sum of the thermal expansion $\omega_{expn}(r)$ (thickening) of the heat exchanger along the z axis and the bending of its middle plane ($z = 0$), $\omega_{bnd}(r)$. We will evaluate the bending by using an approximation of plate theory for a plate with properties which vary through the thickness [5]: the radial coefficient of thermal expansion $\beta_r(z)$ and the modulus of elasticity $E_r(z)$.

According to [4, 5], the so-called thermal force and thermal moment are sources of bending stresses and strains:

$$N_t(r) = \frac{1}{1-\nu}\int_{-H/2}^{H/2}\beta_r(z)E_r(z)T(r,\ z)\,dz,$$
$$M_t(r) = \frac{1}{1-\nu}\int_{-H/2}^{H/2}\beta_r(z)E_r(z)T(r,\ z)\,z\,dz, \qquad (1)$$

which are determined by the temperature field $T(r, z)$ in the metal. Henceforth, the temperature is reckoned from the temperature of the fluid at the inlet of the heat exchanger ($T_{in} = 0$).

After fourfold integration of the equation of thermoelasticity of circular plates [4, 5] over the radius, we obtain

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